Vector Semantics

Introduction

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(borrowing from: Dan Jurafsky and Jim Martin)

Why vector models of meaning? computing the similarity between words

“**fast**” is similar to “**rapid**”

“**tall**” is similar to “**height**”

Question answering:

*Q: “How **tall** is Mt. Everest?”*

*Candidate A: “The official **height** of Mount Everest is 29029 feet”*

Word similarity for plagiarism detection

MAINFRAMES

Mainframes are primarily referred to large computers with rapid, advanced processing capabilities that can execute and perform tasks equivalent to many Personal Computers (PCs) machines networked together. It is characterized with high quantity Random Access Memory (RAM), very large secondary storage devices, and high-speed processors to cater for the needs of the computers under its service.

Consisting of advanced components, mainframes have the capability of running multiple large applications required by many and most enterprises and organizations. This is one of its advantages. Mainframes are also suitable to cater for those applications (programs) or files that are of very high

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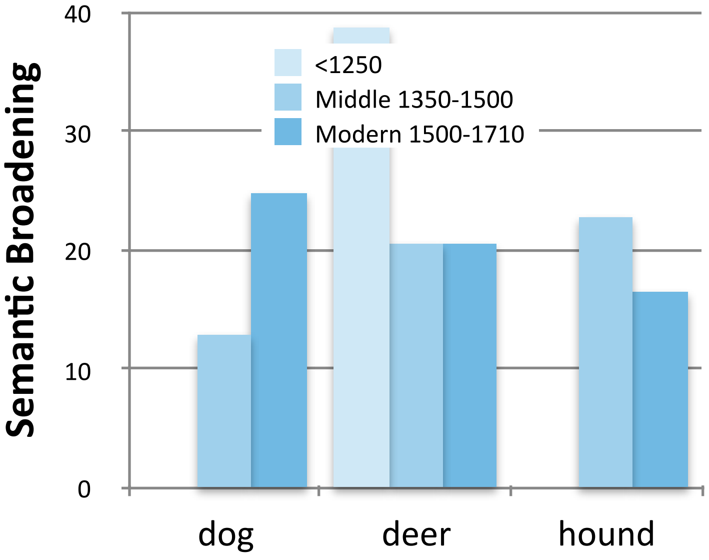
Mainframes usually are referred those computers with fast, advanced processing capabilities that could perform by itself tasks that may require a lot of Personal Computers (PC) Machines. Usually mainframes would have lots of RAMs, very large secondary storage devices, and very fast processors to cater for the needs of those computers under its service.

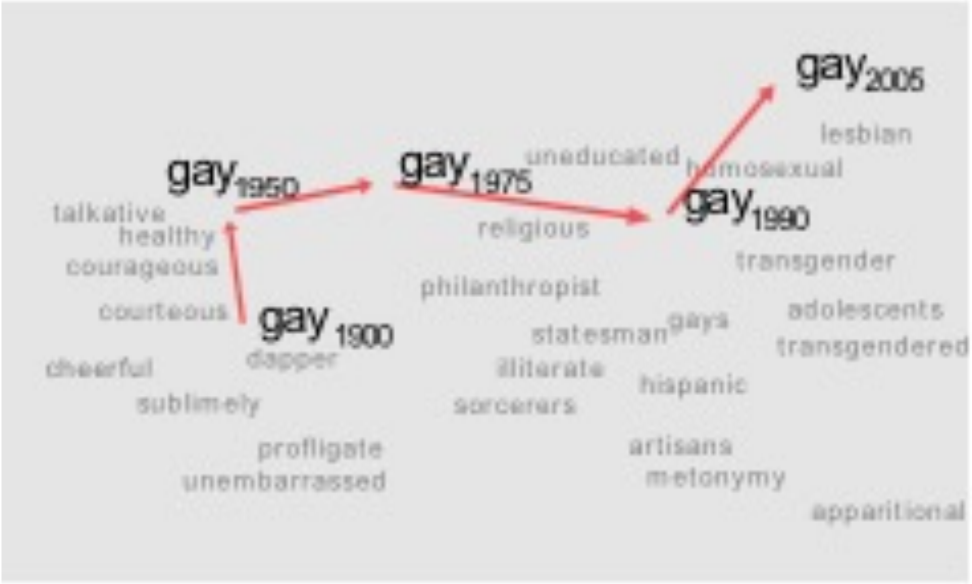
Due to the advanced components mainframes have, these computers have the capability of running multiple large applications required by most enterprises, which is one of its advantage. Mainframes are also suitable to cater for those applications or files that are of very large demand

Word similarity for historical change: semantic change over time

Sagi, Kaufmann Clark 2013



Kulkarni, Al-Rfou, Perozzi, Skiena 2015



Problems with thesaurus-based meaning

- We don’t have a thesaurus for every language
- We can’t have a thesaurus for every year
  - For change detection, we need to compare word meanings in year t to year t+1
- Thesauruses have problems with **recall**
  - Many words and phrases are missing
  - Thesauri work less well for verbs, adjectives

Distributional models of meaning
= vector-space models of meaning
= vector semantics

Intuitions: Zellig Harris (1954):

- "oculist and eye-doctor … occur in almost the same environments"
- "If A and B have almost identical environments we say that they are synonyms."

Firth (1957):

- "You shall know a word by the company it keeps!"

Intuition of distributional word similarity

- Nida example: Suppose I asked you what is ***tesgüino?***

```
A bottle of tesgüino is on the table
Everybody likes tesgüino
Tesgüino makes you drunk
We make tesgüino out of corn.
```

- From context words humans can guess ***tesgüino*** means
  - an alcoholic beverage like beer
- Intuition for algorithm:
  - **Two words are similar if they have similar word contexts.**

Three kinds of vector models

Sparse vector representations

1. Mutual-information weighted word co-occurrence matrices

Dense vector representations:

2. Singular value decomposition (and Latent Semantic Analysis)
3. Neural-network-inspired models (skip-grams, CBOW)

Shared intuition

- Model the meaning of a word by “embedding” in a vector space.
- The meaning of a word is a vector of numbers
  - Vector models are also called “**embeddings**”.
- Contrast: word meaning is represented in many computational linguistic applications by a vocabulary index (“word number 545”)
- Old philosophy joke:

  Q: What’s the meaning of life?

  A: LIFE’

Vector Semantics

Words and co-occurrence vectors

Co-occurrence Matrices

- We represent how often a word occurs in a document
  - **Term-document matrix**
- Or how often a word occurs with another
  - **Term-term matrix**

    (or **word-word co-occurrence matrix**

    or **word-context matrix**)

Term-document matrix

- Each cell: count of word *w* in a document *d*:
  - Each document is a count vector in $\mathbb{N}^V$: a column below

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	1	8	15
soldier	2	2	12	36
fool	37	58	1	5
clown	6	117	0	0

Similarity in term-document matrices

Two documents are similar if their vectors are similar

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	1	8	15
soldier	2	2	12	36
fool	37	58	1	5
clown	6	117	0	0

The words in a term-document matrix

- Each word is a $\boxed{\text{count vector}}$ in $\mathbb{N}^D$: a row below

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	1	8	15
soldier	2	2	12	36
fool	37	58	1	5
clown	6	117	0	0

The words in a term-document matrix

- Two **words** are similar if their vectors are similar

	As You Like It	Twelfth Night	Julius Caesar	Henry V
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clown	6	117	0	0

The word-word or word-context matrix

- Instead of entire documents, use smaller contexts
  - Paragraph
  - Window of $\pm$ 4 words
- A word is now defined by a vector over counts of context words
- Instead of each vector being of length D
- Each vector is now of length |V|
- The word-word matrix is |V|x|V|

Word-Word matrix
Sample contexts ± 7 words

sugar, a sliced lemon, a tablespoonful of	**apricot**	preserve or jam, a pinch each of,
their enjoyment. Cautiously she sampled her first	**pineapple**	and another fruit whose taste she likened
well suited to programming on the digital	**computer**.	In finding the optimal R-stage policy from
for the purpose of gathering data and	**information**	necessary for the study authorized in the

	aardvark	computer	data	pinch	result	sugar	…
apricot	0	0	0	1	0	1	
pineapple	0	0	0	1	0	1	
digital	0	2	1	0	1	0	
information	0	1	6	0	4	0	
…	…						

Word-word matrix

- We showed only 4x6, but the real matrix is 50,000 x 50,000
  - So it's very **sparse**
    - Most values are 0.
  - That's OK, since there are lots of efficient algorithms for sparse matrices.
- The size of windows depends on your goals
  - The shorter the windows , the more **syntactic** the representation

    $\pm$ 1-3 very syntacticy
  - The longer the windows, the more **semantic** the representation

    $\pm$ 4-10 more semanticy

2 kinds of co-occurrence between 2 words

(Schütze and Pedersen, 1993)

- First-order co-occurrence (**syntagmatic association**):
  - They are typically nearby each other.
  - *wrote* is a first-order associate of *book* or *poem*.
- Second-order co-occurrence (**paradigmatic association**):
  - They have similar neighbors.
  - *wrote* is a second- order associate of words like *said* or *remarked*.

Vector Semantics

Positive Pointwise Mutual Information (PPMI)

Problem with raw counts

- Raw word frequency is not a great measure of association between words
  - It’s very skewed
    - “the” and “of” are very frequent, but maybe not the most discriminative
- We’d rather have a measure that asks whether a context word is **particularly informative** about the target word.
  - Positive Pointwise Mutual Information (PPMI)

Pointwise Mutual Information

- **Pointwise mutual information:**

  Do events x and y co-occur more than if they were independent?

$$\text{PMI}(X,Y) = \log_2 \frac{P(x,y)}{P(x)P(y)}$$

PMI between two words: (Church & Hanks 1989)

Do words x and y co-occur more than if they were independent?

$$\text{PMI}(word_1, word_2) = \log_2 \frac{P(word_1, word_2)}{P(word_1)P(word_2)}$$

Positive Pointwise Mutual Information

- PMI ranges from $-\infty$ to $+\infty$
- But the negative values are problematic
  - Things are co-occurring **less than** we expect by chance
  - Unreliable without enormous corpora
    - Imagine w1 and w2 whose probability is each 10^{-6}
    - Hard to be sure p(w1,w2) is significantly different than 10^{-12}
  - Plus it's not clear people are good at "unrelatedness"
- So we just replace negative PMI values by 0
- Positive PMI (PPMI) between word1 and word2:

$$\mathrm{PPMI}(word_1, word_2) = \max\left(\log_2 \frac{P(word_1, word_2)}{P(word_1)P(word_2)}, 0\right)$$

Computing PPMI on a term-context matrix

- Matrix F with W rows (words) and C columns (contexts)
- f_{ij} is # of times w_i occurs in context c_j

	aardvark	computer	data	pinch	result	sugar
apricot	0	0	0	1	0	1
pineapple	0	0	0	1	0	1
digital	0	2	1	0	1	0
information	0	1	6	0	4	0

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}} \qquad p_{i*} = \frac{\sum_{j=1}^{C} f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}} \qquad p_{*j} = \frac{\sum_{i=1}^{W} f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}}$$

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*}\,p_{*j}} \qquad ppmi_{ij} = \begin{cases} pmi_{ij} & \text{if } pmi_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}}$$

Count(w,context)

	computer	data	pinch	result	sugar
apricot	0	0	1	0	1
pineapple	0	0	1	0	1
digital	2	1	0	1	0
information	1	6	0	4	0

$$p(w_i) = \frac{\sum_{j=1}^{C} f_{ij}}{N} \qquad p(c_j) = \frac{\sum_{i=1}^{W} f_{ij}}{N}$$

p(w=information,c=data) = 6/19 = .32

p(w=information) = 11/19 = .58

p(c=data) = 7/19 = .37

p(w,context) / p(w)

	computer	data	pinch	result	sugar	p(w)
apricot	0.00	0.00	0.05	0.00	0.05	0.11
pineapple	0.00	0.00	0.05	0.00	0.05	0.11
digital	0.11	0.05	0.00	0.05	0.00	0.21
information	0.05	0.32	0.00	0.21	0.00	0.58
p(context)	0.16	0.37	0.11	0.26	0.11	

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*} p_{*j}}$$

p(w,context)

	computer	data	pinch	result	sugar	**p(w)**
apricot	0.00	0.00	0.05	0.00	0.05	0.11
pineapple	0.00	0.00	0.05	0.00	0.05	0.11
digital	0.11	0.05	0.00	0.05	0.00	0.21
information	0.05	0.32	0.00	0.21	0.00	0.58
p(context)	0.16	0.37	0.11	0.26	0.11	

- pmi(information,data) = $\log_2$ (.32 / (.37*.58)) = .58

(.57 using full precision)

PPMI(w,context)

	computer	data	pinch	result	sugar
apricot	-	-	2.25	-	2.25
pineapple	-	-	2.25	-	2.25
digital	1.66	0.00	-	0.00	-
information	0.00	0.57	-	0.47	-

Weighting PMI

- PMI is biased toward infrequent events
  - Very rare words have very high PMI values
- Two solutions:
  - Give rare words slightly higher probabilities
  - Use add-delta smoothing (which has a similar effect)

Weighting PMI: Giving rare context words slightly higher probability

- Raise the context probabilities to $\alpha = 0.75$:

$$\mathrm{PPMI}_\alpha(w,c) = \max(\log_2 \frac{P(w,c)}{P(w)P_\alpha(c)}, 0)$$

$$P_\alpha(c) = \frac{count(c)^\alpha}{\sum_c count(c)^\alpha}$$

- This helps because $P_\alpha(c) > P(c)$ for rare c
- Consider two events, P(a) = .99 and P(b)=.01
- $P_\alpha(a) = \frac{.99^{.75}}{.99^{.75}+.01^{.75}} = .97 \quad P_\alpha(b) = \frac{.01^{.75}}{.01^{.75}+.01^{.75}} = .03$

Add-2 Smoothed Count(w,context)

	computer	data	pinch	result	sugar
apricot	2	2	3	2	3
pineapple	2	2	3	2	3
digital	4	3	2	3	2
information	3	8	2	6	2

	p(w,context) [add-2]					p(w)
	computer	data	pinch	result	sugar	
apricot	0.03	0.03	0.05	0.03	0.05	0.20
pineapple	0.03	0.03	0.05	0.03	0.05	0.20
digital	0.07	0.05	0.03	0.05	0.03	0.24
information	0.05	0.14	0.03	0.10	0.03	0.36
p(context)	0.19	0.25	0.17	0.22	0.17	

PPMI versus add-2 smoothed PPMI

PPMI(w,context)

	computer	data	pinch	result	sugar
apricot	-	-	2.25	-	2.25
pineapple	-	-	2.25	-	2.25
digital	1.66	0.00	-	0.00	-
information	0.00	0.57	-	0.47	-

PPMI(w,context) [add-2]

	computer	data	pinch	result	sugar
apricot	0.00	0.00	0.56	0.00	0.56
pineapple	0.00	0.00	0.56	0.00	0.56
digital	0.62	0.00	0.00	0.00	0.00
information	0.00	0.58	0.00	0.37	0.00

Vector Semantics

Measuring similarity: the cosine

Measuring similarity

- Given 2 target words *v* and *w*
- We'll need a way to measure their similarity.
- Most measure of vectors similarity are based on the:
- **Dot product** or **inner product** from linear algebra

$$\text{dot-product}(\vec{v}, \vec{w}) = \vec{v} \cdot \vec{w} = \sum_{i=1}^{N} v_i w_i = v_1 w_1 + v_2 w_2 + \ldots + v_N w_N$$

  - High when two vectors have large values in same dimensions.
  - Low (in fact 0) for **orthogonal vectors** with zeros in complementary distribution

Problem with dot product

$$\text{dot-product}(\vec{v},\vec{w}) = \vec{v}\cdot\vec{w} = \sum_{i=1}^{N} v_i w_i = v_1 w_1 + v_2 w_2 + \ldots + v_N w_N$$

- Dot product is longer if the vector is longer. Vector length:

$$|\vec{v}| = \sqrt{\sum_{i=1}^{N} v_i^2}$$

- Vectors are longer if they have higher values in each dimension
- That means more frequent words will have higher dot products
- That's bad: we don't want a similarity metric to be sensitive to word frequency

Solution: cosine

- Just divide the dot product by the length of the two vectors!

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

- This turns out to be the cosine of the angle between them!

$$\begin{aligned} \vec{a} \cdot \vec{b} &= |\vec{a}||\vec{b}| \cos\theta \\ \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} &= \cos\theta \end{aligned}$$

Cosine for computing similarity

$$\cos(\vec{v},\vec{w}) = \frac{\overbrace{\vec{v}\bullet\vec{w}}^{\text{Dot product}}}{|\vec{v}||\vec{w}|} = \overbrace{\frac{\vec{v}}{|\vec{v}|}\bullet\frac{\vec{w}}{|\vec{w}|}}^{\text{Unit vectors}} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

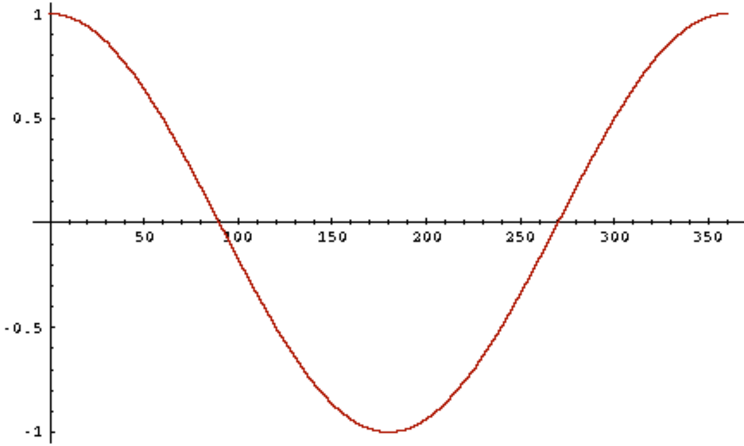
v_i is the PPMI value for word v in context i

w_i is the PPMI value for word w in context i.

Cos($\vec{v}$,$\vec{w}$) is the cosine similarity of $\vec{v}$ and $\vec{w}$

Cosine as a similarity metric

- -1: vectors point in opposite directions
- +1: vectors point in same directions
- 0: vectors are orthogonal



- Raw frequency or PPMI are non-negative, so cosine range 0-1

$$\cos(\vec{v},\vec{w}) = \frac{\vec{v}\bullet\vec{w}}{|\vec{v}||\vec{w}|} = \frac{\vec{v}}{|\vec{v}|}\bullet\frac{\vec{w}}{|\vec{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

	large	data	computer
apricot	2	0	0
digital	0	1	2
information	1	6	1

Which pair of words is more similar?

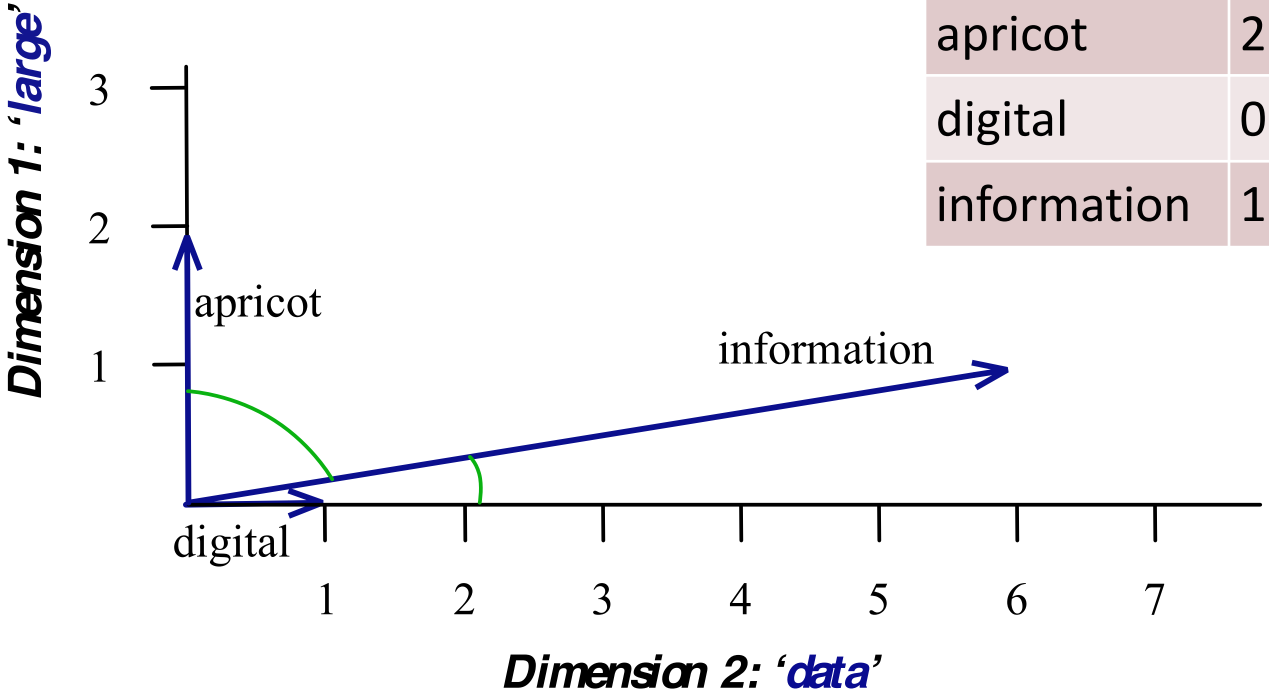
cosine(apricot,information) = $\frac{2+0+0}{\sqrt{2+0+0}\,\sqrt{1+36+1}} = \frac{2}{\sqrt{2}\sqrt{38}} = .23$

cosine(digital,information) = $\frac{0+6+2}{\sqrt{0+1+4}\,\sqrt{1+36+1}} = \frac{8}{\sqrt{38}\sqrt{5}} = .58$

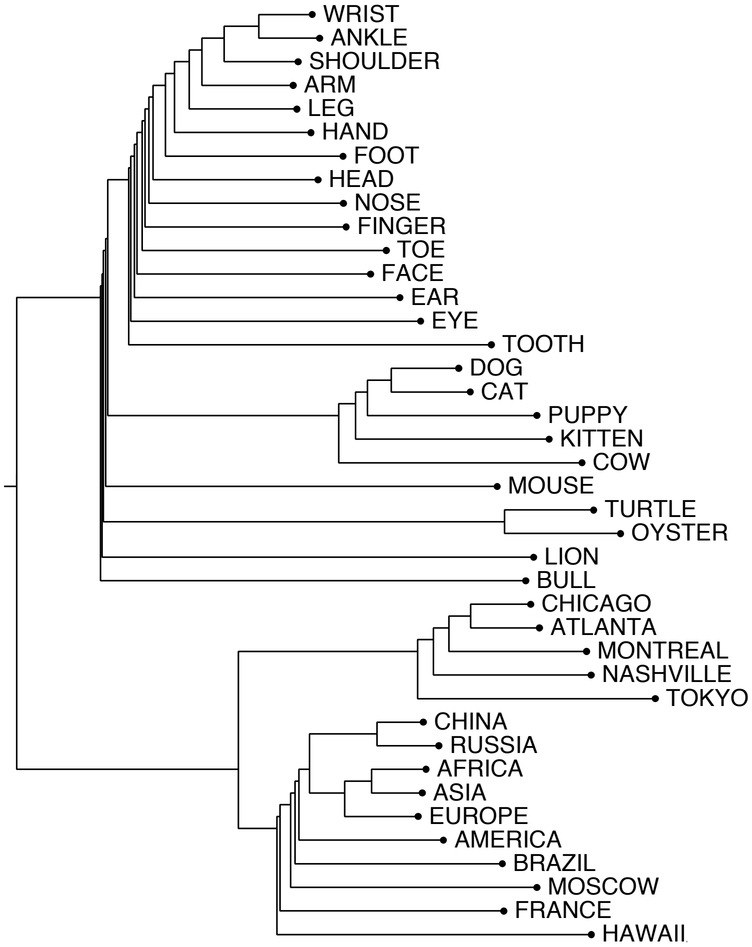
cosine(apricot,digital) = $\frac{0+0+0}{\sqrt{1+0+0}\,\sqrt{0+1+4}} = 0$

Visualizing vectors and angles

	large	data
apricot	2	0
digital	0	1
information	1	6



Clustering vectors to visualize similarity in co-occurrence matrices



Rohde et al. (2006)

Other possible similarity measures

$$\text{sim}_{\text{cosine}}(\vec{v},\vec{w}) = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|} = \frac{\sum_{i=1}^{N} v_i \times w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

$$\text{sim}_{\text{Jaccard}}(\vec{v},\vec{w}) = \frac{\sum_{i=1}^{N} \min(v_i,w_i)}{\sum_{i=1}^{N} \max(v_i,w_i)}$$

$$\text{sim}_{\text{Dice}}(\vec{v},\vec{w}) = \frac{2\times\sum_{i=1}^{N} \min(v_i,w_i)}{\sum_{i=1}^{N}(v_i+w_i)}$$

$$\text{sim}_{\text{JS}}(\vec{v}||\vec{w}) = D(\vec{v}|\frac{\vec{v}+\vec{w}}{2}) + D(\vec{w}|\frac{\vec{v}+\vec{w}}{2})$$

Evaluating similarity (the same as for thesaurus-based)

- Intrinsic Evaluation:
  - Correlation between algorithm and human word similarity ratings
- Extrinsic (task-based, end-to-end) Evaluation:
  - Spelling error detection, WSD, essay grading
  - Taking TOEFL multiple-choice vocabulary tests

```
Levied is closest in meaning to which of these:
    imposed, believed, requested, correlated
```

Alternative to PPMI for measuring association

- **tf-idf** (that's a hyphen not a minus sign)
- The combination of two factors
  - **Term frequency** (Luhn 1957): frequency of the word (can be logged)
  - **Inverse document frequency** (IDF) (Sparck Jones 1972)
    - N is the total number of documents
    - df_i = "document frequency of word *i*"
    - = # of documents with word *I*
  - w_{ij} = *word i in document j*

$$\text{idf}_i = \log\left(\frac{N}{df_i}\right)$$

$$w_{ij} = tf_{ij}\, idf_i$$

tf-idf not generally used for word-word similarity

- But is by far the most common weighting when we are considering the relationship of words to documents

Vector Semantics

Dense Vectors

Sparse versus dense vectors

- PPMI vectors are
  - **long** (length |V|= 20,000 to 50,000)
  - **sparse** (most elements are zero)
- Alternative: learn vectors which are
  - **short** (length 200-1000)
  - **dense** (most elements are non-zero)

Sparse versus dense vectors

- Why dense vectors?
  - Short vectors may be easier to use as features in machine learning (less weights to tune)
  - Dense vectors may generalize better than storing explicit counts
  - They may do better at capturing synonymy:
    - *car* and *automobile* are synonyms; but are represented as distinct dimensions; this fails to capture similarity between a word with *car* as a neighbor and a word with *automobile* as a neighbor

Three methods for getting short dense vectors

- Singular Value Decomposition (SVD)
  - A special case of this is called LSA – Latent Semantic Analysis
- "Neural Language Model"-inspired predictive models
  - skip-grams and CBOW
- Brown clustering

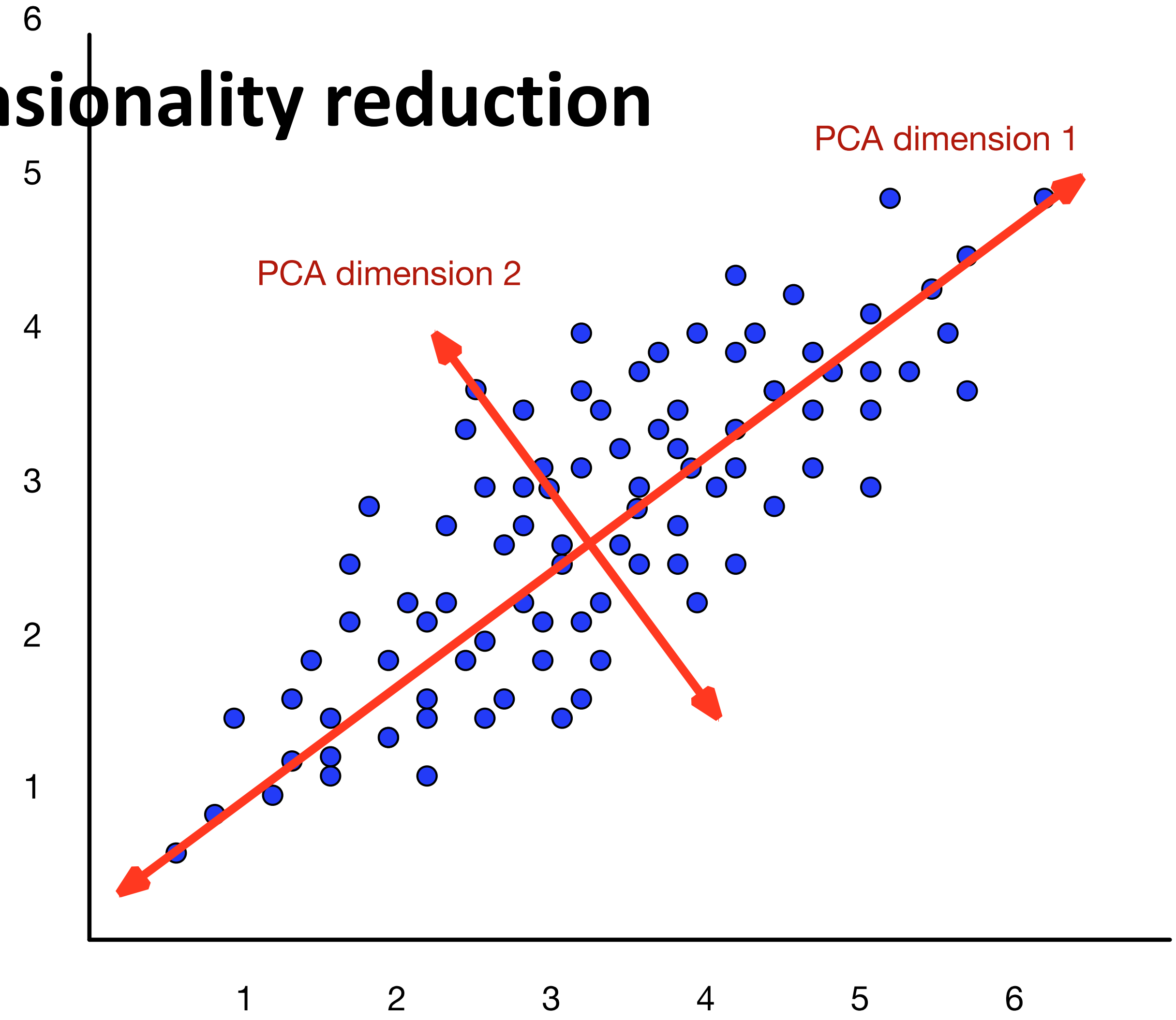
Vector Semantics

Dense Vectors via SVD

Intuition

- Approximate an N-dimensional dataset using fewer dimensions
- By first rotating the axes into a new space
- In which the highest order dimension captures the most variance in the original dataset
- And the next dimension captures the next most variance, etc.
- Many such (related) methods:
  - PCA – principle components analysis
  - Factor Analysis
  - SVD

Dimensionality reduction



Singular Value Decomposition

Any rectangular w x c matrix X equals the product of 3 matrices:

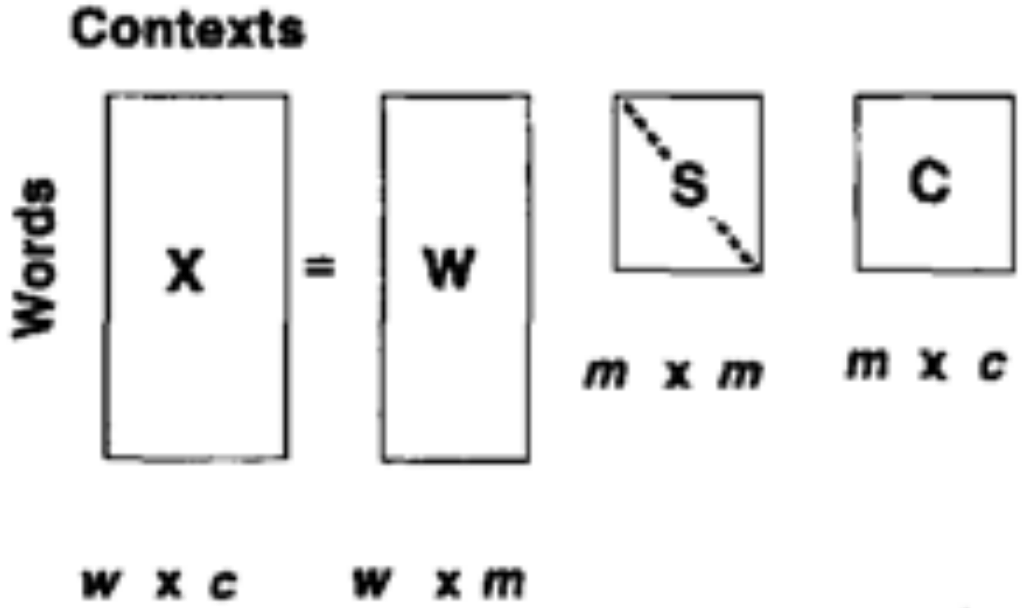
W: rows corresponding to original but m columns represents a dimension in a new latent space, such that

- M column vectors are orthogonal to each other
- Columns are ordered by the amount of variance in the dataset each new dimension accounts for

S: diagonal *m* x *m* matrix of **singular values** expressing the importance of each dimension.

C: columns corresponding to original but m rows corresponding to singular values

Singular Value Decomposition

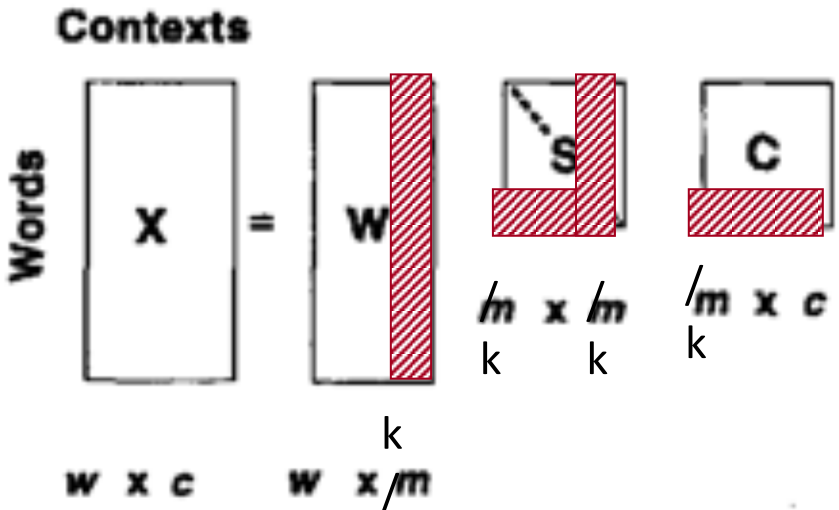


Landuaer and Dumais 1997

SVD applied to term-document matrix: Latent Semantic Analysis

Deerwester et al (1988)

- If instead of keeping all m dimensions, we just keep the top k singular values. Let's say 300.
- The result is a least-squares approximation to the original X



LSA more details

- 300 dimensions are commonly used
- The cells are commonly weighted by a product of two weights
  - Local weight: Log term frequency
  - Global weight: either idf or an entropy measure

Let’s return to PPMI word-word matrices

- Can we apply to SVD to them?

SVD applied to term-term matrix

$$\underset{|V| \times |V|}{\begin{bmatrix} \\ \\ X \\ \\ \\ \end{bmatrix}} = \underset{|V| \times |V|}{\begin{bmatrix} \\ \\ W \\ \\ \\ \end{bmatrix}} \underset{|V| \times |V|}{\begin{bmatrix} \sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_V \end{bmatrix}} \underset{|V| \times |V|}{\begin{bmatrix} \\ \\ C \\ \\ \\ \end{bmatrix}}$$

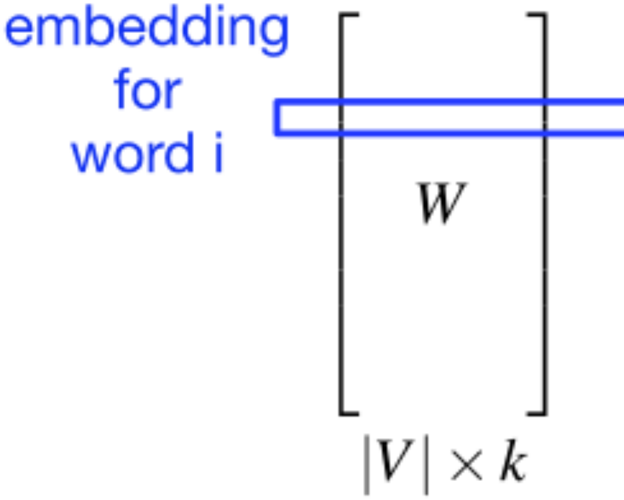
(I'm simplifying here by assuming the matrix has rank |V|)

Truncated SVD on term-term matrix

$$\underset{|V|\times|V|}{\begin{bmatrix} \\ \\ X \\ \\ \\ \end{bmatrix}} = \underset{|V|\times k}{\begin{bmatrix} \\ \\ W \\ \\ \\ \end{bmatrix}} \underset{k\times k}{\begin{bmatrix} \sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_k \end{bmatrix}} \underset{k\times|V|}{\begin{bmatrix} & C & \end{bmatrix}}$$

Truncated SVD produces embeddings

- Each row of W matrix is a k-dimensional representation of each word *w*
- K might range from 50 to 1000
- Generally we keep the top k dimensions, but some experiments suggest that getting rid of the top 1 dimension or even the top 50 dimensions is helpful (Lapesa and Evert 2014).



Embeddings versus sparse vectors

- Dense SVD embeddings sometimes work better than sparse PPMI matrices at tasks like word similarity
  - Denoising: low-order dimensions may represent unimportant information
  - Truncation may help the models generalize better to unseen data.
  - Having a smaller number of dimensions may make it easier for classifiers to properly weight the dimensions for the task.
  - Dense models may do better at capturing higher order co-occurrence.

Vector Semantics

Embeddings inspired by neural language models: skip-grams and CBOW

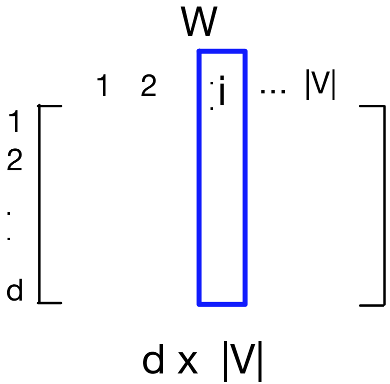
Prediction-based models:
An alternative way to get dense vectors

- **Skip-gram** (Mikolov et al. 2013a) **CBOW** (Mikolov et al. 2013b)
- Learn embeddings as part of the process of word prediction.
- Train a neural network to predict neighboring words
  - Inspired by **neural net language models**.
  - In so doing, learn dense embeddings for the words in the training corpus.
- Advantages:
  - Fast, easy to train (much faster than SVD)
  - Available online in the `word2vec` package
  - Including sets of pretrained embeddings!

Skip-grams

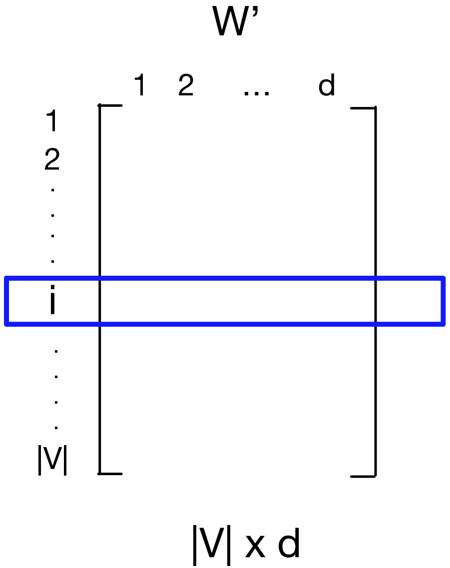
- Predict each neighboring word
  - in a context window of 2*C* words
  - from the current word.
- So for C=2, we are given word w_t and predicting these 4 words: $[w_{t-2}, w_{t-1}, w_{t+1}, w_{t+2}]$

Skip-grams learn 2 embeddings for each w



input embedding *v*, in the input matrix *W*

- Column *i* of the input matrix *W* is the 1×*d* embedding v_i for word *i* in the vocabulary.



output embedding *v*′, in output matrix W'

- Row *i* of the output matrix *W*′ is a *d* × 1 vector embedding v'_i for word *i* in the vocabulary.

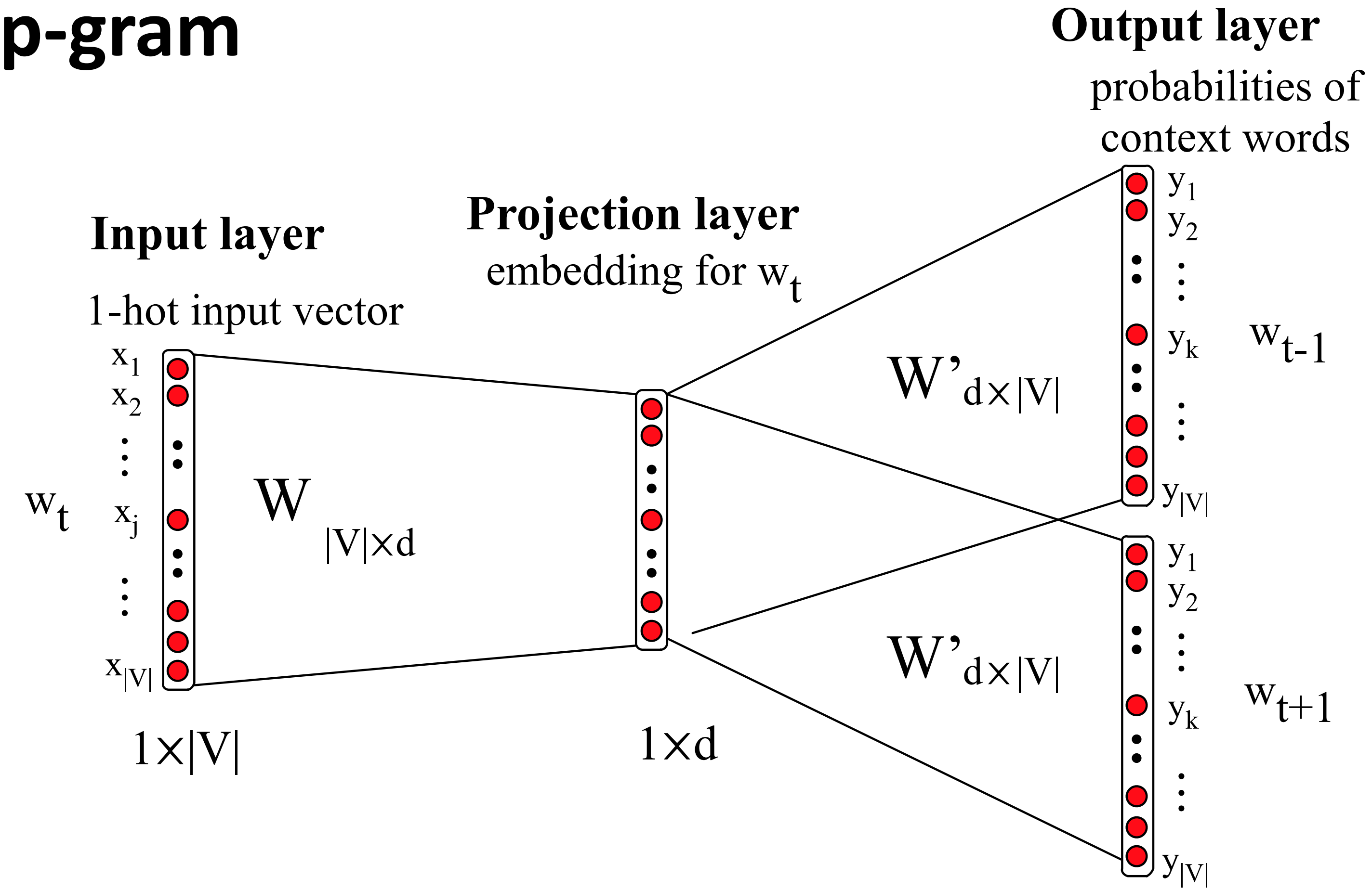
Setup

- Walking through corpus pointing at word $w(t)$, whose index in the vocabulary is j, so we'll call it w_j $(1 < j < |V|)$.
- Let's predict $w(t+1)$, whose index in the vocabulary is k $(1 < k < |V|)$. Hence our task is to compute $P(w_k|w_j)$.

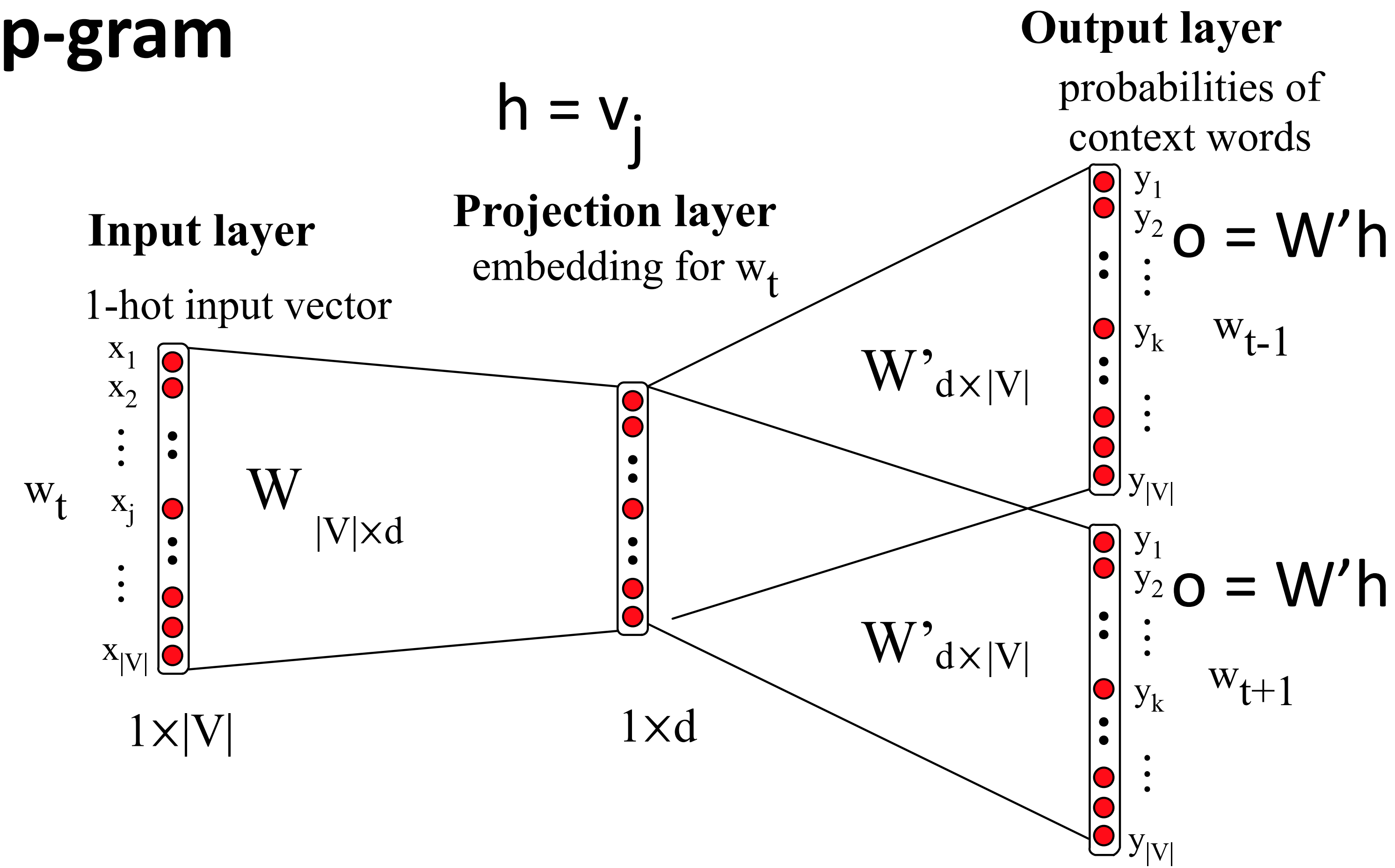
One-hot vectors

- A vector of length |V|
- 1 for the target word and 0 for other words
- So if “popsicle” is vocabulary word 5
- The **one-hot vector** is
- [0,0,0,0,1,0,0,0,0…….0]

Skip-gram

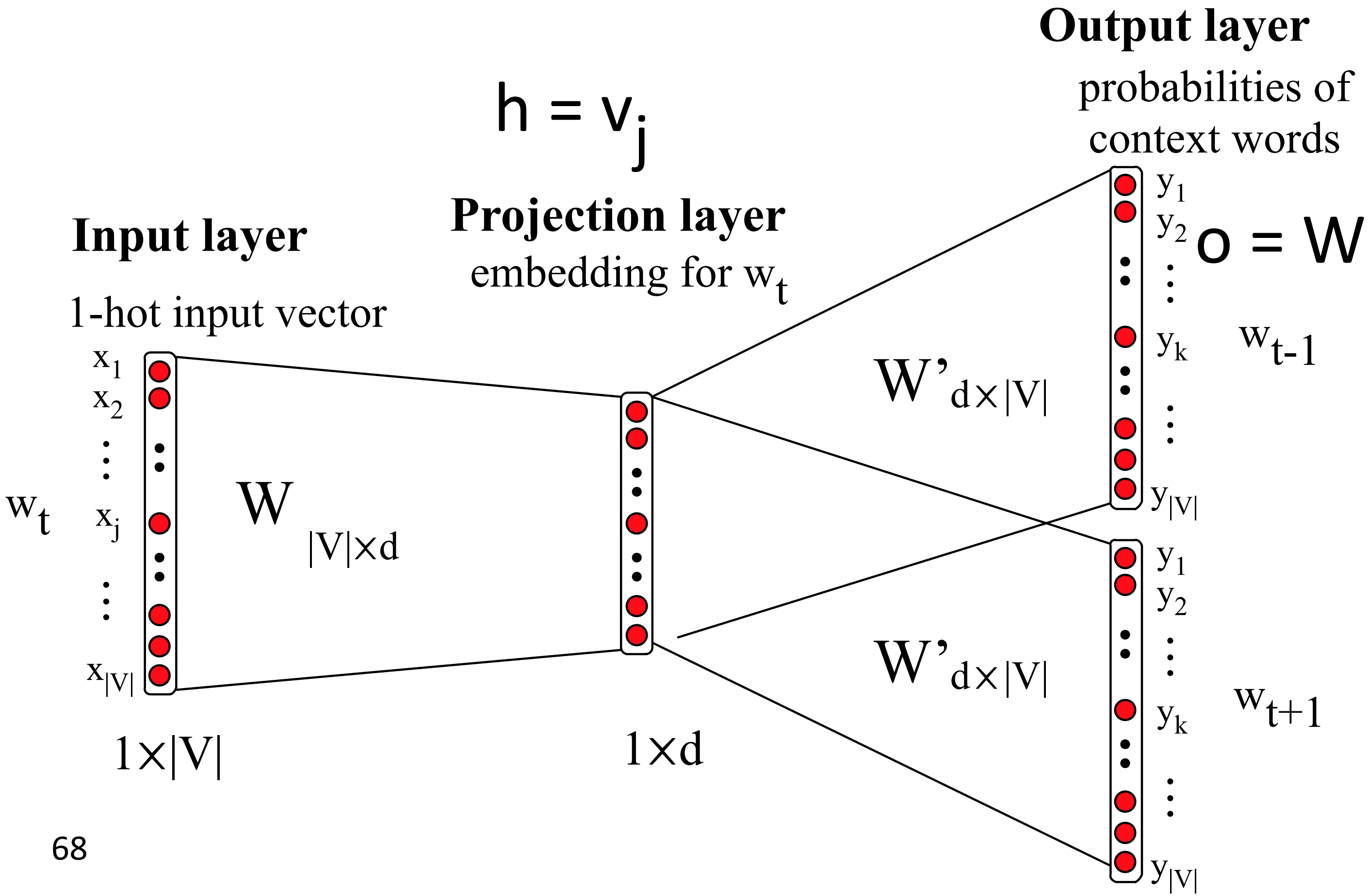

Output layer
probabilities of
context words
Input layer
Projection layer
embedding for w_t
1-hot input vector
x_1
x_2
x_j
$x_{|V|}$
w_t
$W_{|V|\times d}$
$W'_{d\times|V|}$
$W'_{d\times|V|}$
y_1
y_2
y_k
$y_{|V|}$
w_{t-1}
y_1
y_2
y_k
$y_{|V|}$
w_{t+1}
$1\times|V|$
$1\times d$

Skip-gram



$h = v_j$
Output layer
probabilities of context words
Input layer
Projection layer
embedding for w_t
1-hot input vector
x_1
x_2
x_j
$x_{|V|}$
w_t
$W_{|V|\times d}$
$1\times|V|$
$1\times d$
$W'_{d\times|V|}$
$W'_{d\times|V|}$
y_1
y_2
y_k
$y_{|V|}$
$o = W'h$
w_{t-1}
y_1
y_2
y_k
$y_{|V|}$
$o = W'h$
w_{t+1}

Skip-gram



$h = v_j$

$o = W'h$

$o_k = v'_k h$

$o_k = v'_k \cdot v_j$

Turning outputs into probabilities

- $o_k = v'_k \cdot v_j$
- We use softmax to turn into probabilities

$$p(w_k|w_j) = \frac{exp(v'_k \cdot v_j)}{\sum_{w' \in |V|} exp(v'_w \cdot v_j)}$$

Embeddings from W and W'

- Since we have two embeddings, v_j and v'_j for each word w_j
- We can either:
  - Just use v_j
  - Sum them
  - Concatenate them to make a double-length embedding

But wait; how do we learn the embeddings?

$$\underset{\theta}{\operatorname{argmax}} \; \log \; p(\text{Text})$$

$$\underset{\theta}{\operatorname{argmax}} \log \prod_{t=1}^{T} p(w^{(t-C)}, \ldots, w^{(t-1)}, w^{(t+1)}, \ldots, w^{(t+C)})$$

$$\underset{\theta}{\operatorname{argmax}} \sum_{-c \leq j \leq c, j \neq 0} \log p(w^{(t+j)} | w^{(t)})$$

$$= \underset{\theta}{\operatorname{argmax}} \sum_{t=1}^{T} \sum_{-c \leq j \leq c, j \neq 0} \log \frac{exp(v'^{(t+j)} \cdot v^{(t)})}{\sum_{w \in |V|} exp(v'_w \cdot v^{(t)})}$$

$$= \underset{\theta}{\operatorname{argmax}} \sum_{t=1}^{T} \sum_{-c \leq j \leq c, j \neq 0} \left[v'^{(t+j)} \cdot v^{(t)} - \log \sum_{w \in |V|} exp(v'_w \cdot v^{(t)}) \right]$$

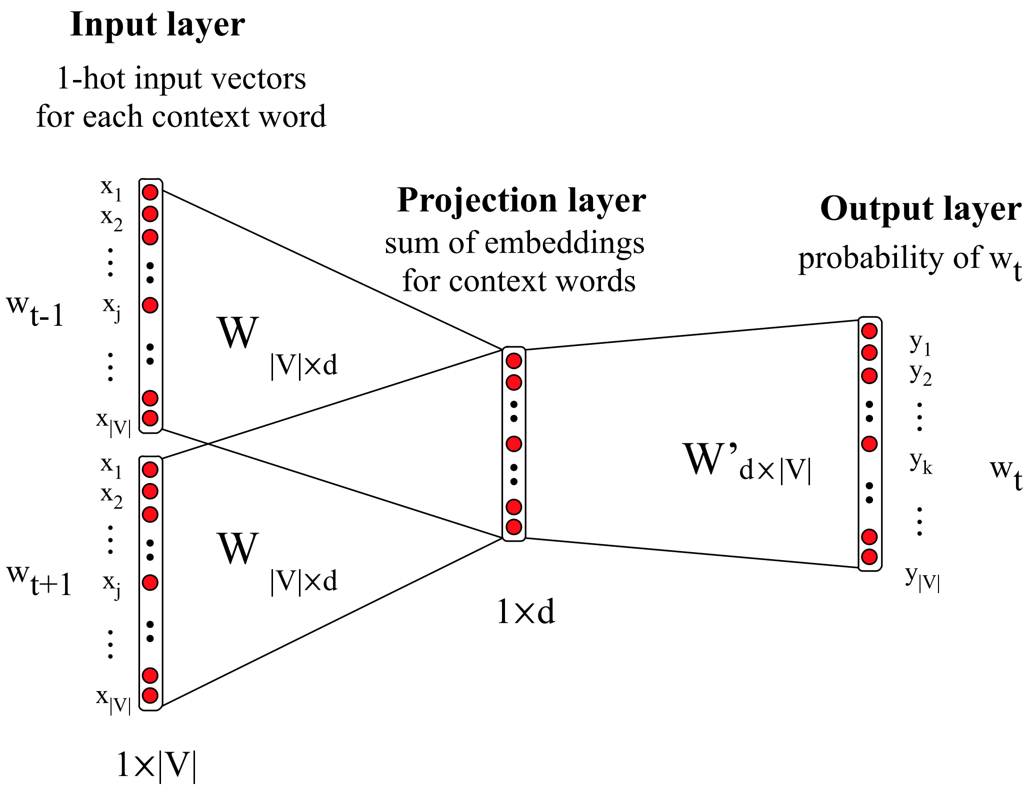
Relation between skipgrams and PMI!

- If we multiply WW'^T
- We get a |V|x|V| matrix M , each entry m_{ij} corresponding to some association between input word i and output word j
- Levy and Goldberg (2014b) show that skip-gram reaches its optimum just when this matrix is a shifted version of PMI:

$$WW'^T = M^{\mathrm{PMI}} - \log k$$

- So skip-gram is implicitly factoring a shifted version of the PMI matrix into the two embedding matrices.

CBOW (Continuous Bag of Words)



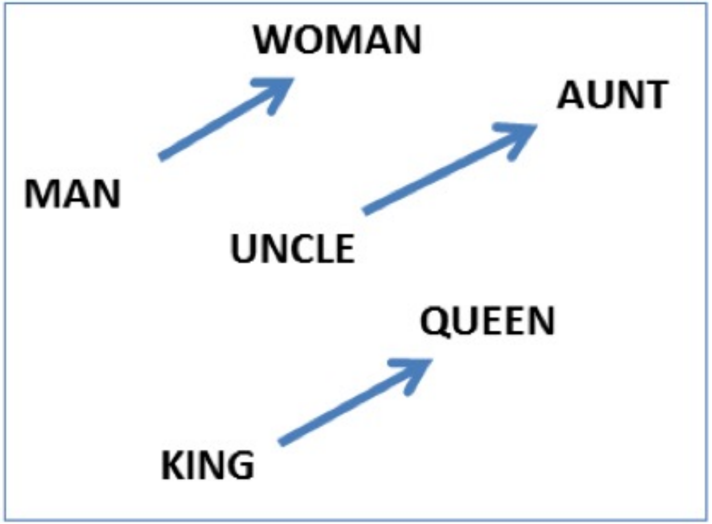
Properties of embeddings

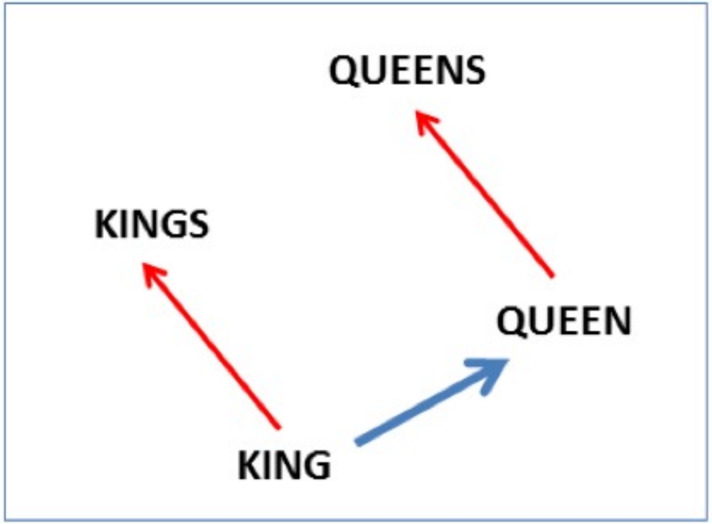
- Nearest words to some embeddings (Mikolov et al. 20131)

target:	Redmond	Havel	ninjutsu	graffiti	capitulate
	Redmond Wash.	Vaclav Havel	ninja	spray paint	capitulation
	Redmond Washington	president Vaclav Havel	martial arts	grafitti	capitulated
	Microsoft	Velvet Revolution	swordsmanship	taggers	capitulating

Embeddings capture relational meaning!

- vector(*'king'*) - vector(*'man'*) + vector(*'woman'*) ≈ vector('queen')
  vector(*'Paris'*) - vector(*'France'*) + vector(*'Italy'*) ≈ vector('Rome')





Vector Semantics

Evaluating similarity

Evaluating similarity

- Extrinsic (task-based, end-to-end) Evaluation:
  - Question Answering
  - Spell Checking
  - Essay grading
- Intrinsic Evaluation:
  - Correlation between algorithm and human word similarity ratings
    - Wordsim353: 353 noun pairs rated 0-10. *sim(plane,car)=5.77*
  - Taking TOEFL multiple-choice vocabulary tests
    - `Levied is closest in meaning to:`

      `imposed, believed, requested, correlated`

Summary

- Distributional (vector) models of meaning
  - **Sparse** (PPMI-weighted word-word co-occurrence matrices)
  - **Dense**:
    - Word-word SVD 50-2000 dimensions
    - Skip-grams and CBOW (embeddings available in word2vec)

A great semantic vector space for documents

- words have low-dimensional embeddings, useful for many computational linguistic applications
- documents are a weighted combination of words
- documents as a vector in the low-dimensional space
- this allows
  - semantic document clustering (k-means, hierarchical, etc.)
  - search for similar documents (prior art in patents, etc.)